Learning-Algorithms from Bayesian Principles

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The Goal of My Research

"To understand the fundamental principles of learning from data and use them to develop algorithms that can learn like living beings."

Human Learning: At the age of 6 months.



Converged at the age of 12 months



Transfer Knowledge at the age of 14 months



Human learning \neq Deep learning Continual & incremental Bulk

My current research focuses on reducing this gap!

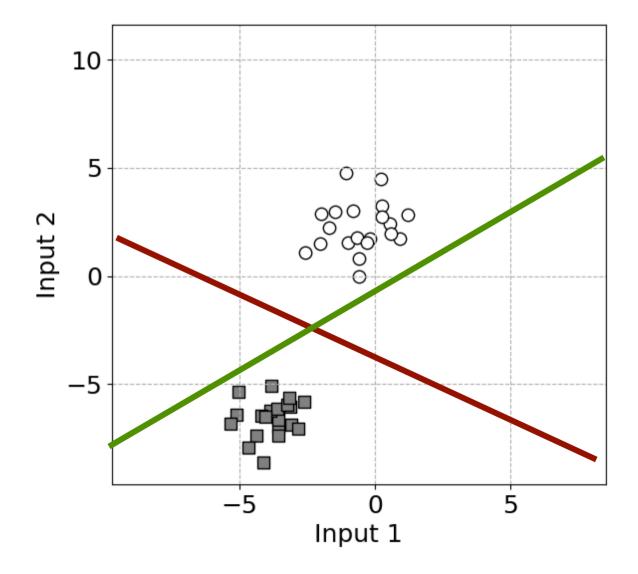
Continual lifelong learning with NN (Parisi et al. 2019)

Learning-Algorithms from Bayesian Principles

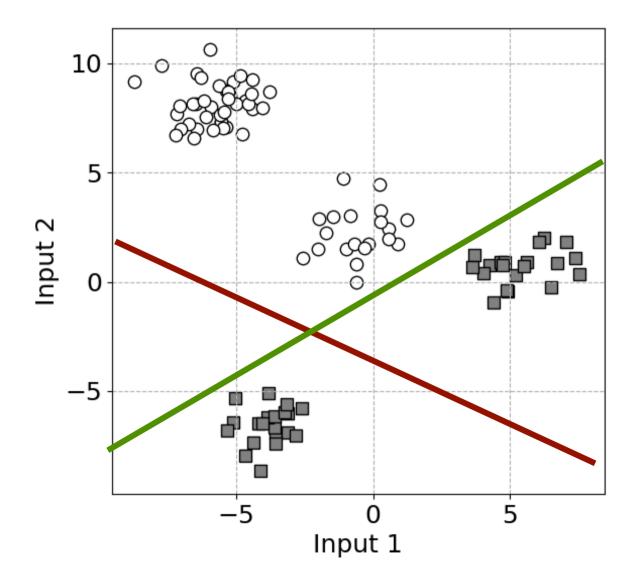
- Bayesian principles to
 - Derive existing learning-algorithms
 - Classical (least-squares, Newton, HMM, Kalman...)
 - Deep Learning (SGD, RMSprop, Adam)
 - Improve them
 - Uncertainty and data-relevance for free
 - Design new ones
 - Continual Learning with Bayes
- Impact: Everything with one common principle

Why Bayes?

Which is a good classifier?

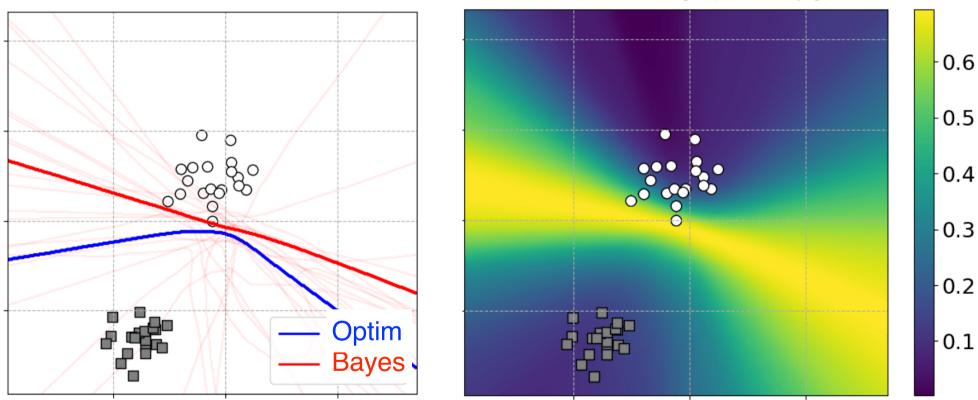


Which is a good classifier?



"What the model does not know but should know": Knowledge gap

The Bayesian Solution



Uncertainty (Entropy)

(By Kazuki Osawa) <u>https://github.com/team-approx-bayes/dl-with-bayes</u>

Bayesian Learning Rule

Learning algorithms from Bayes Uncertainty for free

Learning-Algorithms by BayeSian Principles Alstats 2017

Learning by optimization: $\theta \leftarrow \theta - \rho H^{-1} \nabla_{\theta} \ell(\theta)$

Learning by Bayes:
$$\lambda \leftarrow (1 - \rho)\lambda - \rho \nabla_{\mu} \mathbb{E}_q \left[\ell(\theta) \right]$$

 $\{V^{-1}m, V^{-1}\}$

 $\{\mathbb{E}(\theta), \mathbb{E}(\theta\theta^{\top})\}\$

Natural parameters

Expectation/moment/

mean parameters

Natural and Expectation parameters of q

e.g., Gaussian distribution

$$q(\theta) := \mathcal{N}(\theta|m, V)$$

$$\exp\left[m^{\top}V^{-1}\theta - \frac{1}{2}\theta^{\top}V^{-1}\theta\right]$$

Learning by Bayes

Learning by optimization: $\theta \leftarrow \theta - \rho H^{-1} \nabla_{\theta} \ell(\theta)$

Learning by Bayes:
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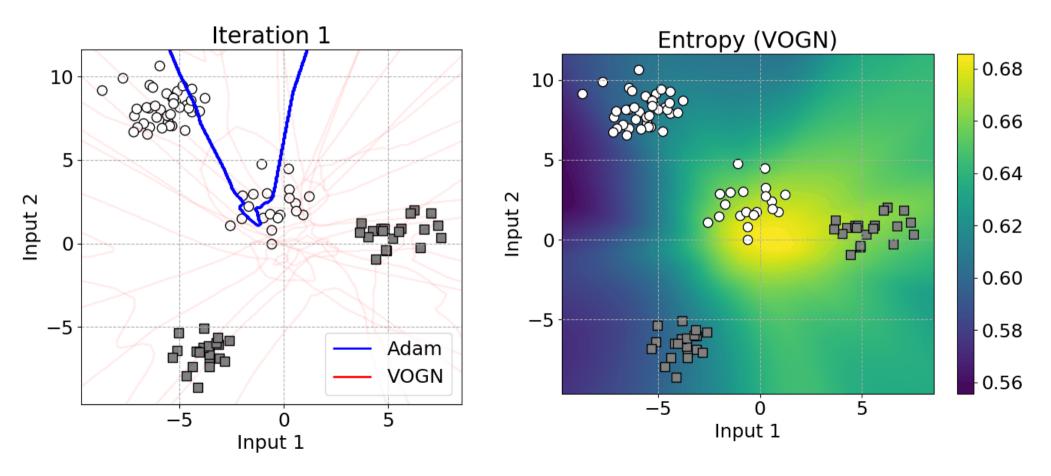
Natural and Expectation parameters of q

Alstats 2017 – ICML 2017 Classical algorithms: Least-squares, Newton's method, Kalman filters, Baum-Welch, Forward-backward, etc.
 Bayesian inference: EM, Laplace's method, SVI, VMP. ICML 2018 - Deep learning: SGD, RMSprop, Adam. NeurIPS 2018 – — Reinforcement learning: parameter-space exploration, natural policy-search. ISITA 2018 ICLR 2018 - Continual learning: Elastic-weight consolidation. - Online learning: Exponential-weight average. NIPS 2017 – Global optimization: Natural evolutionary strategies, Gaussian homotopy, continuation method & smoothed optimization. List incomplete...

Learning by Bayes for Deep Learning

ICML 2018, NeurIPS 2019

The VOGN algorithm for fast uncertainty in deep learning.



Optimization -> Bayes

NeurIPS 2019

Switching from "Adam" to "VOGN" in two lines of code change.

```
import torch
+import torchsso
train_loader = torch.utils.data.DataLoader(train_dataset)
model = MLP()
-optimizer = torch.optim.Adam(model.parameters())
+optimizer = torchsso.optim.VOGN(model, dataset_size=len(train_loader.dataset))
```

Available at https://github.com/team-approx-bayes/dl-with-bayes

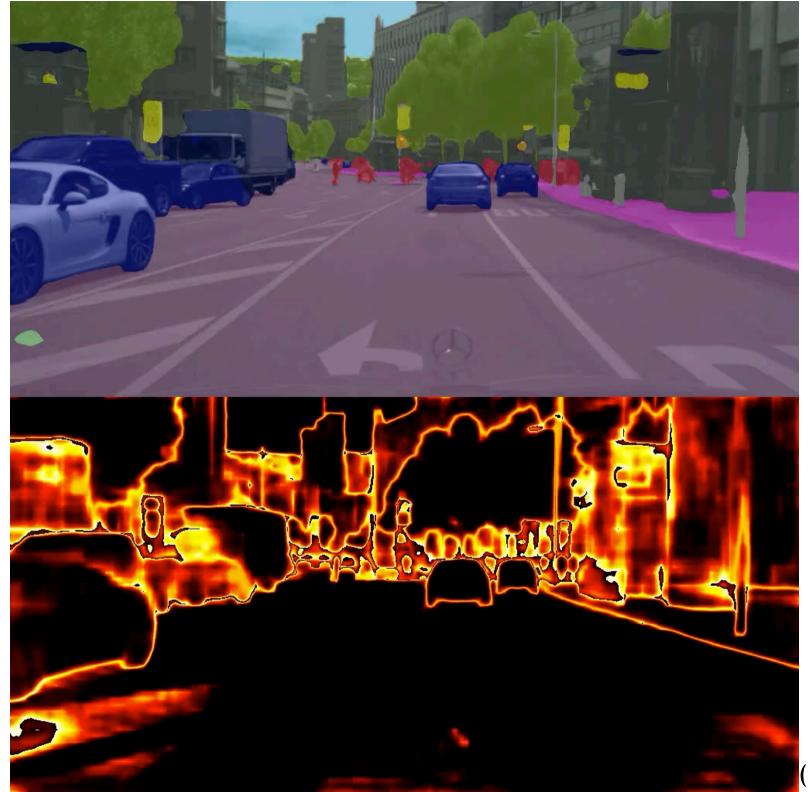


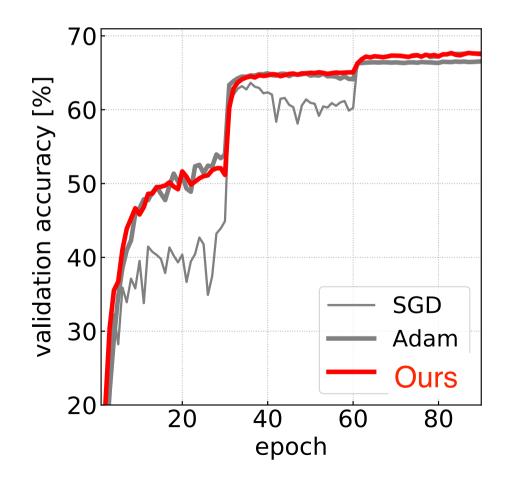
Image Segmentation

Uncertainty (entropy of class probs)

(By Roman Bachmann)¹⁷

NeurIPS 2019 Practical DL with Bayes (on ImageNet)

State-of-the-art performance and convergence rate, while preserving benefits of Bayesian principles ("well-calibrated" uncertainty).



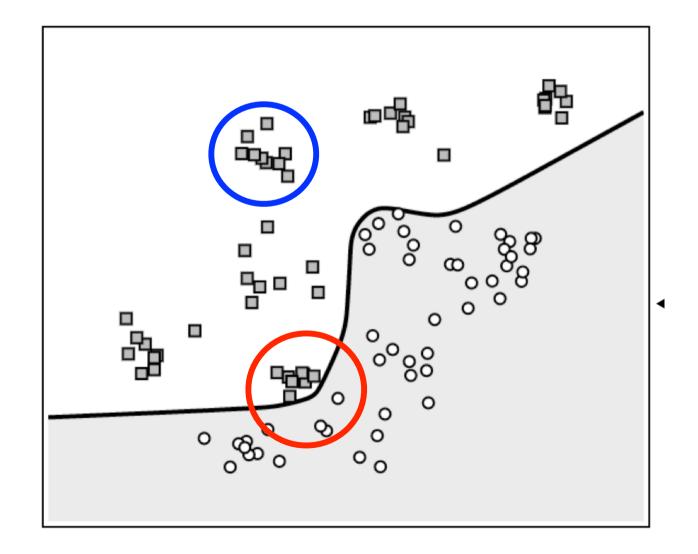
Accuracy

A New Bayesian Principle

"Data relevance" for free

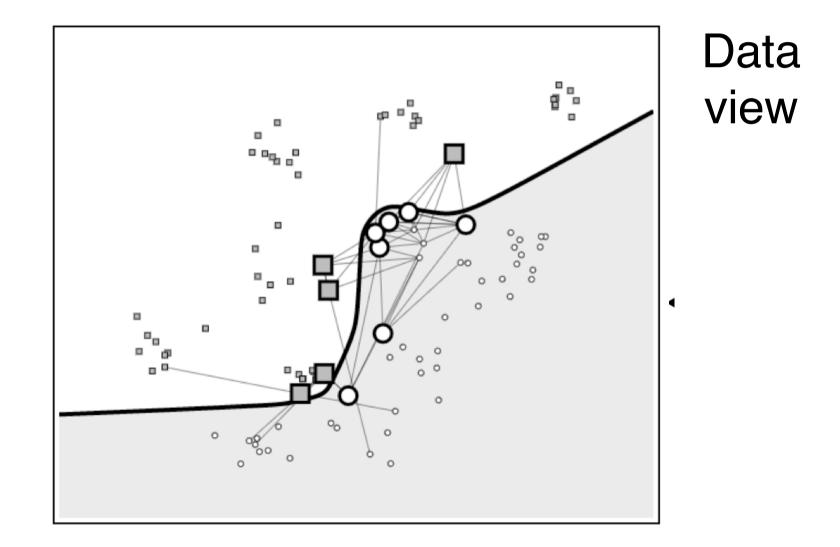
Defining Relevance

Which examples are most important for the classifier? Red vs Blue.



Model view vs Data view

Bayesian principles "automatically" define data-relevance.



Statistics > Machine Learning

Search...

Help | Adva

Approximate Inference Turns Deep Networks into Gaussian Processes

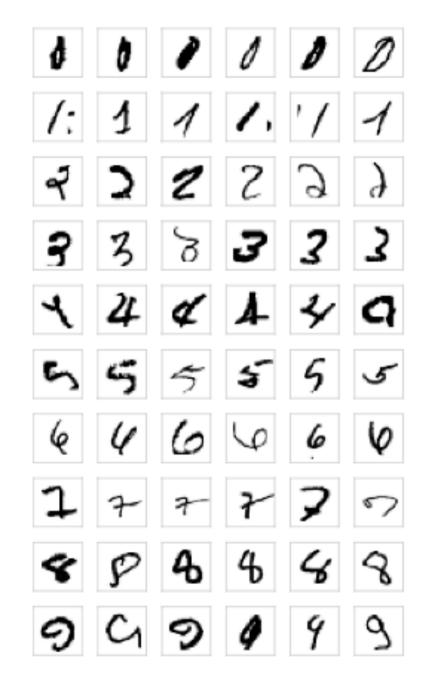
Mohammad Emtiyaz Khan, Alexander Immer, Ehsan Abedi, Maciej Korzepa

(Submitted on 5 Jun 2019)

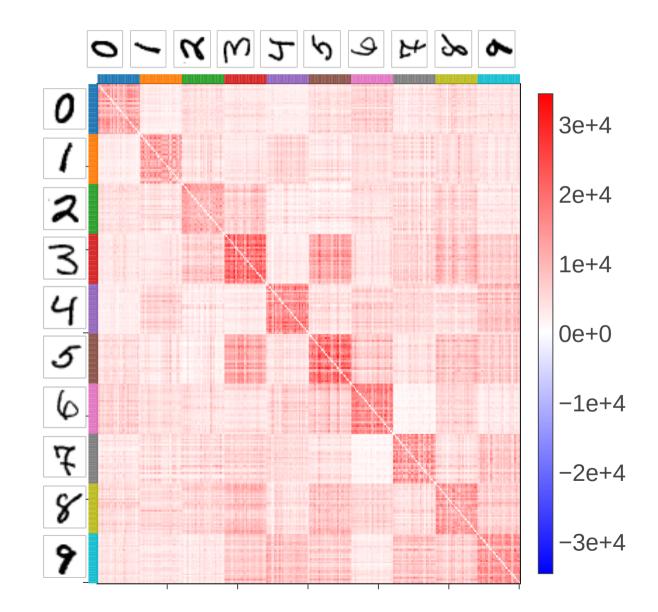
Deep neural networks (DNN) and Gaussian processes (GP) are two powerful models with several theoretical connections relating them, but the relationship between their training methods is not well understood. In this paper, we show that certain Gaussian posterior approximations for Bayesian DNNs are equivalent to GP posteriors. As a result, we can obtain a GP kernel and a nonlinear feature map simply by training the DNN. Surprisingly, the resulting kernel is the neural tangent kernel which has desirable theoretical properties for infinitely-wide DNNs. We show feature maps obtained on real datasets and demonstrate the use of the GP marginal likelihood to tune hyperparameters of DNNs. Our work aims to facilitate further research on combining DNNs and GPs in practical settings.



Relevant



Similarity (Kernel) Matrix



Bayesian Duality Principle (WIP)

$$\sum_{i=1}^{N} \frac{\text{likelihood}}{\rho_{i}(x_{i})} + \gamma \theta^{\top} \theta$$

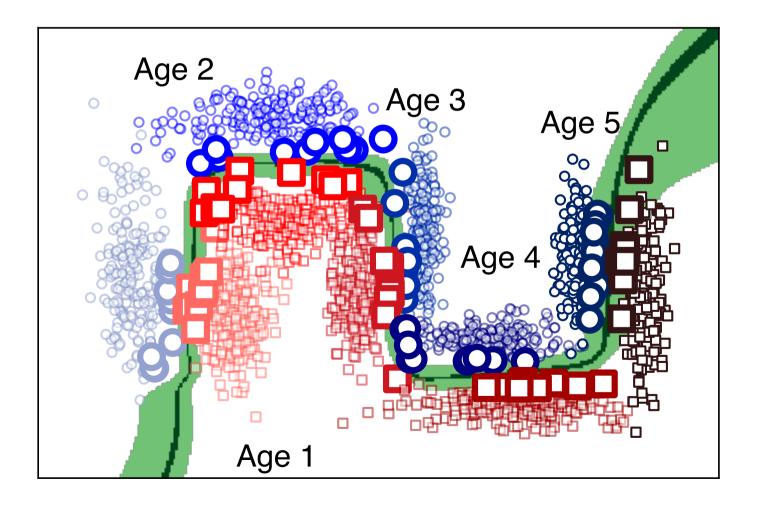
$$\approx \sum_{i=1}^{N} \frac{1}{\sigma_{i}^{2}} [\tilde{y}_{i} - \phi_{i}(x_{i})^{\top} \theta]^{2} + \gamma \theta^{\top} \theta$$

$$\uparrow$$

"Dual" variables

Deep Continual Learning with Bayes

FROMP: Functional Regularization of Memorable Past "Identify, Memorize, and Regularize"



Works Well on Standard Benchmarks

On MNIST and CIFAR-100, we get state-of-the-art results!

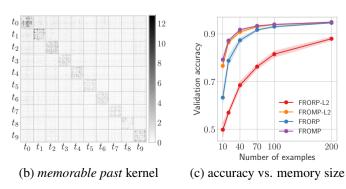
Method	Permuted MNIST	Split MNIST
DLP (Smola et al., 2003)	82%	61.2%
EWC (Kirkpatrick et al., 2017)	84%	63.1%
SI (Zenke et al., 2017)	86%	98.9%
Improved VCL (Swaroop et al., 2019)	$93\% \pm 1$	$98.4\% \pm 0.4$
+ random Coreset	94.6 % \pm 0.3 (200 p/t)	$98.2\% \pm 0.4 \ (40 \text{ p/t})$
FRCL-RND (Titsias et al., 2019)	$94.2\% \pm 0.1 \ (200 \text{ p/t})$	$96.7\% \pm 1.0 (40 \text{ p/t})$
FRCL-TR (Titsias et al., 2019)	$94.3\% \pm 0.1 \ (200 \text{ p/t})$	$97.4\% \pm 0.6 (40 \text{ p/t})$
FRORP-L ₂	$87.9\% \pm 0.7 (200 \text{ p/t})$	$98.5\% \pm 0.2 (40 \text{ p/t})$
FROMP- L_2	$94.6\% \pm 0.1 \ (200 \text{ p/t})$	$98.7\% \pm 0.1 \; (40 \text{ p/t})$
FRORP	$94.6\% \pm 0.1 \ (200 \text{ p/t})$	99.0 % ± 0.1 (40 p/t)
FROMP	94.9 % \pm 0.1 (200 p/t)	99.0 % \pm 0.1 (40 p/t)

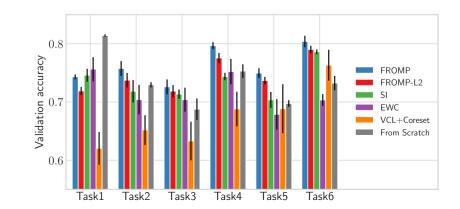
FRORP-L2

200

FROMP-L2 - FRORP

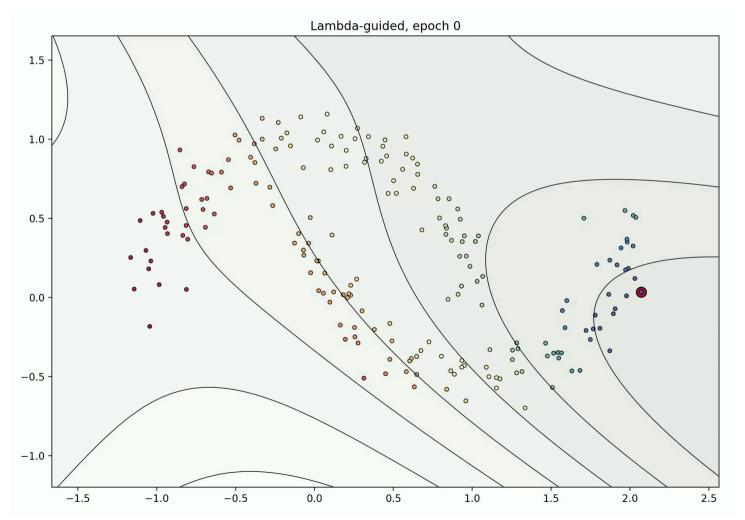
FROMP





Relevance of Examples

Given a minibatch at each iteration, we select examples with less noise (low variance of epsilon_i in the approximated linear model).



(By Roman Bachmann)

Summary

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References

Available at https://emtiyaz.github.io/publications.html

Conjugate-Computation Variational Inference : Converting Variational Inference in Non-Conjugate Models to Inferences in Conjugate Models, (AISTATS 2017) M.E. KHAN AND W. LIN [Paper] [Code for Logistic Reg

Fast and Scalable Bayesian Deep Learning by Weight-Perturbation in Adam, (ICML 2018) M.E. KHAN, D. NIELSEN, V. TANGKARATT, W. LIN, Y. GAL, AND A. SRIVASTAVA, [ArXiv Version] [Code] [Slides]

Practical Deep Learning with Bayesian Principles, (Under review) K. Osawa, S. Swaroop, A. Jain, R. Eschenhagen, R.E. Turner, R. Yokota, M.E. Khan. [arXiv]

Approximate Inference Turns Deep Networks into Gaussian Processes, (Under review) M.E. Khan, A. Immer, E. Abedi, M. Korzepa. [arXiv]

Learning-Algorithms from Bayesian Principles

A long paper to be released before my NeurIPS tutorial

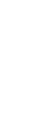
Mon Dec 9th 08:30 -- 10:30 AM @ West Hall A **Deep Learning with Bayesian Principles** Mohammad Emtiyaz Khan

L Emtiyaz Khan »

Deep learning and Bayesian learning are considered two entirely different fields often used in complementary settings. It is clear that combining ideas from the two fields would be beneficial, but how can we achieve this given their fundamental differences?

This tutorial will introduce modern Bayesian principles to bridge this gap. Using these principles, we can derive a range of learning-algorithms as special cases, e.g., from classical algorithms, such as linear regression and forward-backward algorithms, to modern deep-learning algorithms, such as SGD, RMSprop and Adam. This view then enables new ways to improve aspects of deep learning, e.g., with uncertainty, robustness, and interpretation. It also enables the design of new methods to tackle challenging problems, such as those arising in active learning, continual learning, reinforcement learning, etc.

Overall, our goal is to bring Bayesians and deep-learners closer than ever before, and motivate them to work together to solve challenging real-world problems by combining their strengths.





Havard Rue (KAUST)

Tutorial

A 5 page review

Fast yet Simple Natural-Gradient Descent for Variational Inference in Complex Models

Mohammad Emtiyaz Khan RIKEN Center for Advanced Intelligence Project Tokyo, Japan emtiyaz.khan@riken.jp Didrik Nielsen RIKEN Center for Advanced Intelligence Project Tokyo, Japan didrik.nielsen@riken.jp

Abstract-Bayesian inference plays an important role in advancing machine learning, but faces computational challenges when applied to complex models such as deep neural networks. Variational inference circumvents these challenges by formulating Bayesian inference as an optimization problem and solving it using gradient-based optimization. In this paper, we argue in favor of natural-gradient approaches which, unlike their gradientbased counterparts, can improve convergence by exploiting the information geometry of the solutions. We show how to derive fast yet simple natural-gradient updates by using a duality associated with exponential-family distributions. An attractive feature of these methods is that, by using natural-gradients, they are able to extract accurate local approximations for individual model components. We summarize recent results for Bayesian deep learning showing the superiority of natural-gradient approaches over their gradient counterparts.

Index Terms—Bayesian inference, variational inference, natural gradients, stochastic gradients, information geometry, exponential-family distributions, nonconjugate models. prove the rate of convergence [7]–[9]. Unfortunately, these approaches only apply to a restricted class of models known as *conditionally-conjugate* models, and do not work for nonconjugate models such as Bayesian neural networks.

This paper discusses some recent methods that generalize the use of natural gradients to such large and complex nonconjugate models. We show that, for exponential-family approximations, a duality between their natural and expectation parameter-spaces enables a simple natural-gradient update. The resulting updates are equivalent to a recently proposed method called Conjugate-computation Variational Inference (CVI) [10]. An attractive feature of the method is that it naturally obtains *local* exponential-family approximations for individual model components. We discuss the application of the CVI method to Bayesian neural networks and show some recent results from a recent work [11] demonstrating



Emtiyaz Khan: Fast yet Simple Natural-Gradient Descent for Variational Inference

Acknowledgements

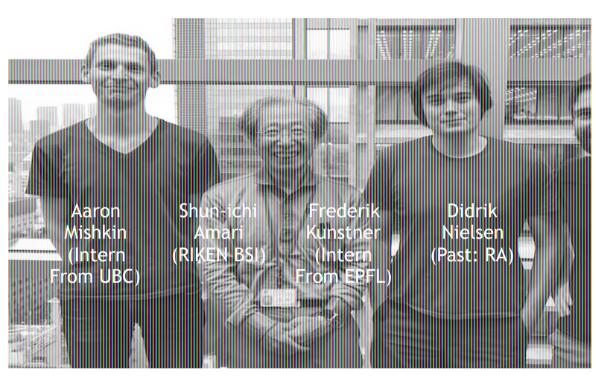
Slides, papers, & code are at emtiyaz.github.io



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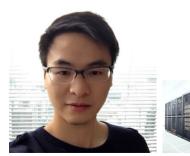


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Kazuki Osawa (Tokyo Tech)

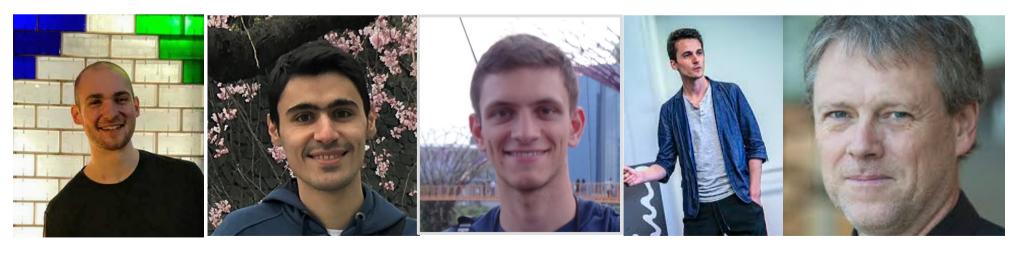
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Approximate Bayesian Inference Team

Looking for interns, research assistants, post-docs, and collaborators

