



The Bayesian Learning Rule for Adaptive Al

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Thanks to Dharmesh Tailor, Siddharth Swaroop, and Thomas Moellenhoff for their help in the preparation of the talk







Al that learn like humans

Quickly adapt to learn new skills, throughout their lives

Human Learning at the age of 6 months.



Converged at the age of 12 months



Transfer skills at the age of 14 months



Fail because too quick to adapt

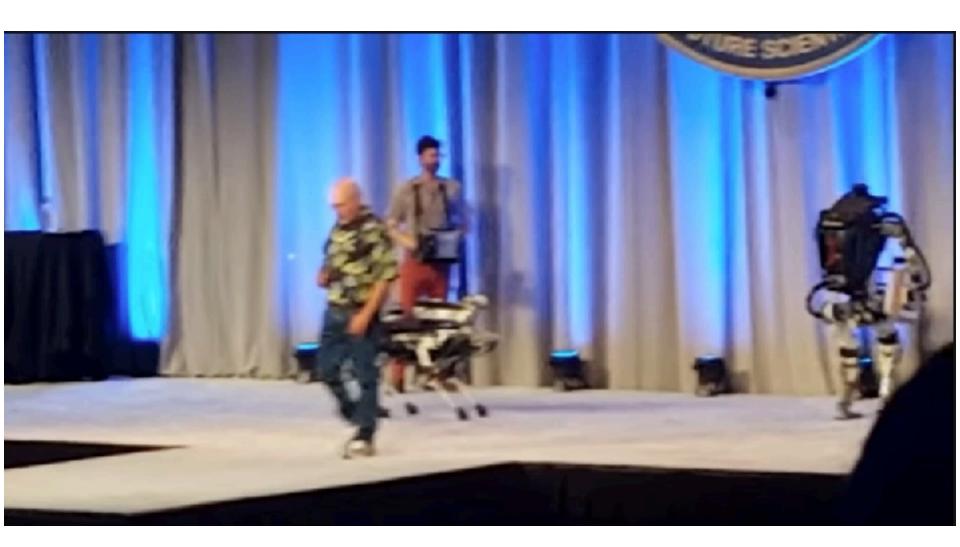
TayTweets: Microsoft AI bot manipulated into being extreme racist upon release

Posted Fri 25 Mar 2016 at 4:38am, updated Fri 25 Mar 2016 at 9:17am



TayTweets is programmed to converse like a teenage girl who has "zero chill", according to Microsoft. (Twitter TayTweets)

Fail because too slow to adapt



Adaptive & Robust Learning with Bayes

- "Good" algorithms are inherently Bayesian
- Bayesian learning rule [1]
- Robustness: Memorable experiences [2]
- Adaptation: Knowledge-Adaptation Priors
 [3,4,5]
- Take away: A new perspective of Bayes, essential for adaptive and robust deep learning

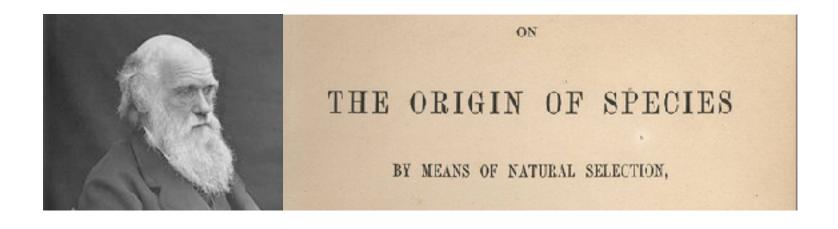
^{1.} Khan and Rue, The Bayesian Learning Rule, arXiv, https://arxiv.org/abs/2107.04562, 2021

^{2.} Tailor, Chang, Swaroop, Solin, Khan. Memorable experiences of ML models (in preparation)

^{3.} Khan et al. Approximate Inference Turns Deep Networks into Gaussian Process, NeurIPS, 2019

^{4.} Pan et al. Continual Deep Learning by Functional Regularisation of Memorable Past, NeurIPS, 2020

^{5.} Khan and Swaroop. Knowledge-Adaptation Priors, NeurIPS, 2021 (https://arxiv.org/abs/2106.08769)



The Origin of Algorithms

A good algorithm must revise its *past* beliefs by using useful *future* information



The Bayesian Learning Rule

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Abstract

We show that many machine-learning algorithms are specific instances of a single algorithm called the *Bayesian learning rule*. The rule, derived from Bayesian principles, yields a wide-range of algorithms from fields such as optimization, deep learning, and graphical models. This includes classical algorithms such as ridge regression, Newton's method, and Kalman filter, as well as modern deep-learning algorithms such as stochastic-gradient descent, RMSprop, and Dropout. The key idea in deriving such algorithms is to approximate the posterior using candidate distributions estimated by using natural gradients. Different candidate distributions result in different algorithms and further approximations to natural gradients give rise to variants of those algorithms. Our work not only unifies, generalizes, and improves existing algorithms, but also helps us design new ones.

Bayesian learning rule

Learning Algorithm	Posterior Approx.	Natural-Gradient Approx.	Sec.				
Optimization Algorithms							
Gradient Descent	Gaussian (fixed cov.)	Delta method	1.3				
Newton's method	Gaussian		1.3				
Multimodal optimization (New)	Mixture of Gaussians	"	3.2				
Deep-Learning Algorithms							
Stochastic Gradient Descent	Gaussian (fixed cov.)	Delta method, stochastic approx.	4.1				
RMSprop/Adam	Gaussian (diagonal cov.)	Delta method, stochastic approx., Hessian approx., square-root scal- ing, slow-moving scale vectors	4.2				
Dropout	Mixture of Gaussians	Delta method, stochastic approx., responsibility approx.	4.3				
STE	Bernoulli	Delta method, stochastic approx.	4.5				
Online Gauss-Newton (OGN) (New)	Gaussian (diagonal cov.)	Gauss-Newton Hessian approx. in Adam & no square-root scaling	4.4				
Variational OGN (New)	"	Remove delta method from OGN	4.4				
BayesBiNN (New)	Bernoulli	Remove delta method from STE	4.5				
Approximate Bayesian Inference Algorithms							
Conjugate Bayes	Exp-family	Set learning rate $\rho_t = 1$	5.1				
Laplace's method	Gaussian	Delta method	4.4				
Expectation-Maximization	Exp- $Family + Gaussian$	Delta method for the parameters	5.2				
Stochastic VI (SVI)	Exp-family (mean-field)	Stochastic approx., local $\rho_t = 1$	5.3				
VMP	"	$ \rho_t = 1 \text{ for all nodes} $	5.3				
Non-Conjugate VMP	"	"	5.3				
Non-Conjugate VI (New)	Mixture of Exp-family	None	5.4				

A Bayesian Origin

$$\min_{\theta} \ \ell(\theta) \qquad \text{vs} \quad \min_{q \in \mathcal{Q}} \ \mathbb{E}_{q(\theta)}[\ell(\theta)] - \mathcal{H}(q)$$

$$\text{Entropy}$$

$$\text{Posterior approximation (expo-family)}$$

Bayesian Learning Rule [1,2] (natural-gradient descent)

Natural and Expectation parameters of q

$$\begin{split} \lambda \leftarrow \dot{\lambda} - \rho \nabla_{\mu}^{\downarrow} \Big\{ \mathbb{E}_q[\ell(\theta)] - \mathcal{H}(q) \Big\} \\ \lambda \leftarrow (1 - \rho) \underline{\lambda} - \rho \nabla_{\mu} \mathbb{E}_q[\ell(\theta)] \\ \text{Old belief} \quad \text{New information = natural gradients} \end{split}$$

Using posterior's information geometry to balance new vs old information

- 1. Khan and Rue, The Bayesian Learning Rule, arXiv, https://arxiv.org/abs/2107.04562, 2021
- 2. Khan and Lin. "Conjugate-computation variational inference...." Alstats (2017).

Bayesian learning rule: $\lambda \leftarrow (1 - \rho)\lambda - \rho \nabla_{\mu} \mathbb{E}_q[\ell(\theta)]$

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Gradient Descent from Bayes

GD:
$$\theta \leftarrow \theta - \rho \nabla_{\theta} \ell(\theta)$$

BLR:
$$m \leftarrow m - \rho \nabla_m \ell(m)$$

"Global" to "local" (the delta method)

$$\mathbb{E}_{q}[\ell(\theta)] \approx \ell(m)$$

$$m \leftarrow m - \rho \nabla_{\mathbf{m}} \mathbb{E}_q[\ell(\theta)]$$

$$\mathbb{E}_{q}[\ell(\theta)] \approx \ell(m) \qquad \lambda \leftarrow \lambda - \rho \nabla_{\mu} \left(\mathbb{E}_{q}[\ell(\theta)] - \mathcal{H}(q) \right)$$

Derived by choosing Gaussian with fixed covariance

Gaussian distribution $q(\theta) := \mathcal{N}(m, 1)$

Natural parameters

Expectation parameters $\mu := \mathbb{E}_q[\theta] = m$

 $\mathcal{H}(q) := \log(2\pi)/2$ **Entropy**

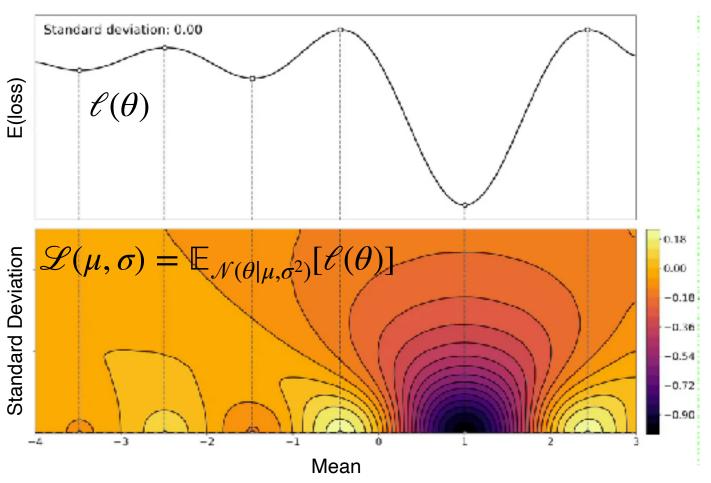
Bayesian learning rule: $\lambda \leftarrow (1-\rho)\lambda - \rho \nabla_{\mu} \mathbb{E}_q[\ell(\theta)]$

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Put the expectation (Bayes) back in!

- 1. Khan, et al. "Fast and scalable Bayesian deep learning by weight-perturbation in Adam." ICML (2018).
- 2. Osawa et al. "Practical Deep Learning with Bayesian Principles." NeurIPS (2019).
- 3. Lin et al. "Handling the positive-definite constraints in the BLR." ICML (2020).

Bayes Objective



Instead of the original loss, optimize a different one (Gaussian convolution)

A popular idea of "implicit regularization" in DL [4], but also common in other fields (RL, search, robust optimization)

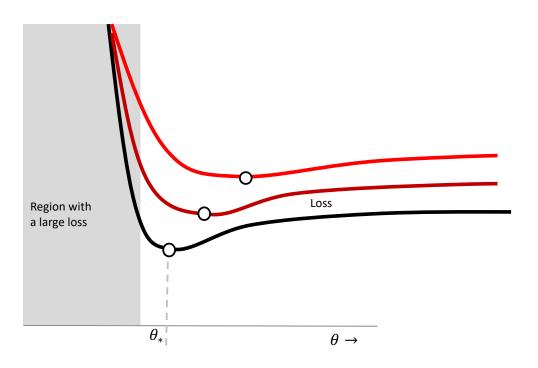
- 1. Zellner, A. "Optimal information processing and Bayes's theorem." *The American Statistician* (1988)
- 2. Many other: Bissiri, et al. (2016), Shawe-Taylor and Williamson (1997), Cesa-Bianchi and Lugosi (2006)
- 3. Huszar's blog, Evolution Strategies, Variational Optimisation and Natural ES (2017)
- 4. Smith et al., On the Origin of Implicit Regularization in Stochastic Gradient Descent, ICLR, 2021

Bayes Prefers Flatter directions

GD: $\theta \leftarrow \theta - \rho \nabla_{\theta} \ell(\theta) \implies \nabla_{\theta} \ell(\theta_*) = 0$

 $\mathsf{BLR:} \quad m \leftarrow m - \rho \nabla_{\mathbf{m}} \mathbb{E}_q[\ell(\theta)] \quad \Longrightarrow \ \nabla_m \mathbb{E}_{q_*}[\ell(\theta)] = 0$

Bayesian solution injects "noise" which has a similar regularization effect to noise in Stochastic GD. It prefers "flatter" directions.



Deriving Learning-Algorithms from the Bayesian Learning Rule

Posterior Approximation \longleftrightarrow Learning-Algorithm



Newton's Method from Bayes

Newton's method: $\theta \leftarrow \theta - H_{\theta}^{-1} \left[\nabla_{\theta} \ell(\theta) \right]$

$$Sm \leftarrow (1-\rho)Sm - \rho \nabla_{\mathbb{E}_{q}(\theta)}\mathbb{E}_{q}[\ell(\theta)]$$

$$-\frac{1}{2}S \leftarrow (1(1-\rho)S)\frac{1}{2}Sp2\nabla\rho\nabla_{\mathbb{E}_{q}(\theta)}\mathbb{E}_{q}[\ell(\theta)]$$

$$\lambda \leftarrow \lambda 1 - \rho \text{Im}_{\mu} \mathbb{E}_{q} \mathbb{V}(\theta)_{q} \mathbb{E}_{q} [\ell(\theta)](q)) \qquad \boxed{-\nabla_{\mu} \mathcal{H}(q) = \lambda}$$

Derived by choosing a multivariate Gaussian

 $\begin{array}{ll} \text{Gaussian distribution} & q(\theta) := \mathcal{N}(\theta|m,S^{-1}) \\ \text{Natural parameters} & \lambda := \{Sm,-S/2\} \\ \text{Expectation parameters} & \mu := \{\mathbb{E}_q(\theta),\mathbb{E}_q(\theta\theta^\top)\} \end{array}$

Newton's Method from Bayes

Newton's method: $\theta \leftarrow \theta - H_{\theta}^{-1} \left[\nabla_{\theta} \ell(\theta) \right]$

Set
$$\rho$$
 =1 to get $m \leftarrow m - H_m^{-1}[\nabla_m \ell(m)]$

$$m \leftarrow m - \rho S^{-1} \nabla_m \ell(m)$$
$$S \leftarrow (1 - \rho)S + \rho H_m$$

Delta Method $\mathbb{E}_q[\ell(\theta)] \approx \ell(m)$

Express in terms of gradient and Hessian of loss:

$$\nabla_{\mathbb{E}_q(\theta)} \mathbb{E}_q[\ell(\theta)] = \mathbb{E}_q[\nabla_{\theta} \ell(\theta)] - 2\mathbb{E}_q[H_{\theta}] m$$

$$\nabla_{\mathbb{E}_q(\theta\theta^\top)}\mathbb{E}_q[\ell(\theta)] = \mathbb{E}_q[H_{\theta}]$$

$$Sm \leftarrow (1 - \rho)Sm - \rho \nabla_{\mathbb{E}_{q}(\theta)} \mathbb{E}_{q}[\ell(\theta)]$$
$$S \leftarrow (1 - \rho)S - \rho 2 \nabla_{\mathbb{E}_{q}(\theta\theta^{\top})} \mathbb{E}_{q}[\ell(\theta)]$$

BLR Variants

RMSprop

Variational Online Gauss-Newton (VOGN)

$$g \leftarrow \hat{\nabla}\ell(\theta)$$

$$s \leftarrow (1 - \rho)s + \rho g^{2}$$

$$\theta \leftarrow \theta - \alpha(\sqrt{s} + \delta)^{-1}g$$

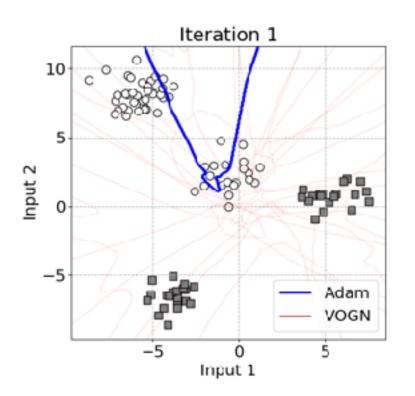
$$g \leftarrow \hat{\nabla}\ell(\theta)$$
, where $\theta \sim \mathcal{N}(m, \sigma^2)$
 $s \leftarrow (1 - \rho)s + \rho(\Sigma_i g_i^2)$
 $m \leftarrow m - \alpha(s + \gamma)^{-1} \nabla_{\theta}\ell(\theta)$
 $\sigma^2 \leftarrow (s + \gamma)^{-1}$

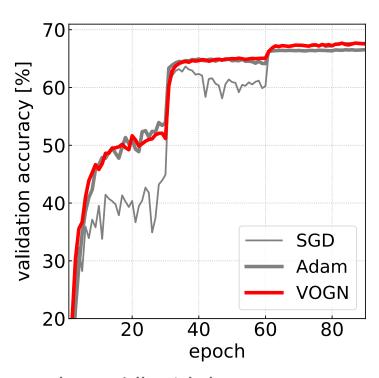
Available at https://github.com/team-approx-bayes/dl-with-bayes

- 1. Khan, et al. "Fast and scalable Bayesian deep learning by weight-perturbation in Adam." *ICML* (2018).
- 2. Osawa et al. "Practical Deep Learning with Bayesian Principles." NeurIPS (2019).
- 3. Lin et al. "Handling the positive-definite constraints in the BLR." ICML (2020).

Uncertainty of Deep Nets

VOGN: A modification of Adam but match the performance on ImageNet





Code available at https://github.com/team-approx-bayes/dl-with-bayes

- 1. Khan, et al. "Fast and scalable Bayesian deep learning by weight-perturbation in Adam." *ICML* (2018).
- 2. Osawa et al. "Practical Deep Learning with Bayesian Principles." NeurIPS (2019).

BLR variant [3] got 1st prize in NeurIPS 2021 Approximate Inference Challenge

Watch Thomas Moellenhoff's talk at https://www.youtube.com/watch?v=LQInIN5EU7E.

Mixture-of-Gaussian Posteriors with an Improved Bayesian Learning Rule

Thomas Möllenhoff¹, Yuesong Shen², Gian Maria Marconi¹ Peter Nickl¹, Mohammad Emtiyaz Khan¹











1 Approximate Bayesian Inference Team RIKEN Center for Al Project, Tokyo, Japan

2 Computer Vision Group Technical University of Munich, Germany

Dec 14th, 2021 — NeurIPS Workshop on Bayesian Deep Learning

- 1. Khan, et al. "Fast and scalable Bayesian deep learning by weight-perturbation in Adam." *ICML* (2018).
- 2. Osawa et al. "Practical Deep Learning with Bayesian Principles." NeurIPS (2019).
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Bayes leads to robust solutions

Avoiding sharp minima

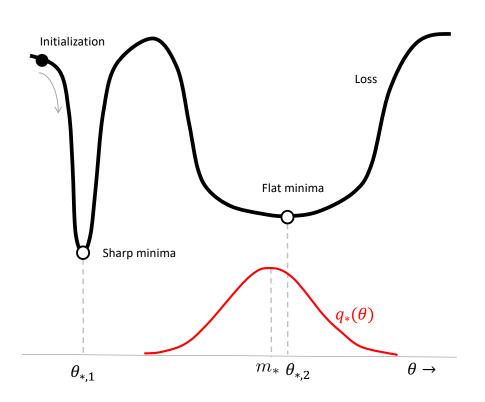
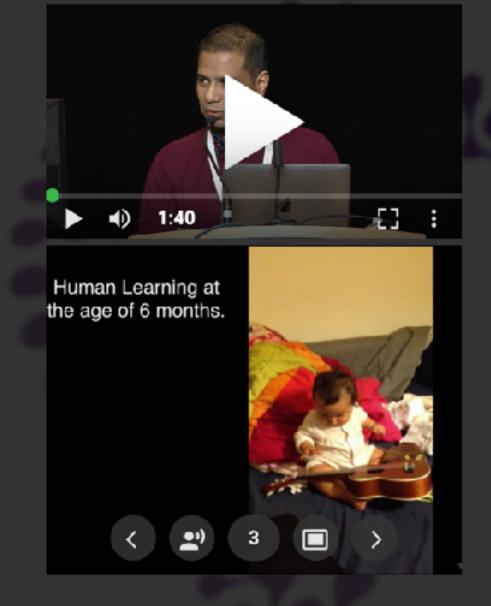




Image Segmentation

Uncertainty (entropy of class probs)

(By Roman Bachmann)25



Deep Learning with Bayesian Principles

by Mohammad Emtiyaz Khan · Dec 9, 2019

NeurIPS 2019 Tutorial

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Robustness

Good algorithms can tell apart relevant vs irrelevant information

How do adapt the knowledge?
Perturbation, Sensitivity, and Duality



BLR Solutions & Their Duality

$$\ell(\theta) = \sum_{i=0}^{N} \ell_i(\theta) \qquad \lambda \leftarrow (1-\rho)\lambda - \sum_{i=0}^{N} \rho \nabla_{\mu} \mathbb{E}_q[\ell_i(\theta)]$$

$$\lambda^* = \sum_{i=0}^{N} \nabla_{\mu^*} \mathbb{E}_{q^*} [-\ell_i(\theta)]$$

Global and local natural parameter

Local parameters are Lagrange Multipliers, measuring the sensitivity of BLR solutions to local perturbation [1]. They can be used to tell apart relevant vs irrelevant data.

Memorable Experiences

MNIST FMNIST 6 T-shirt Pullover SandalAnkle boot Shirt Easy

Outliers

Jncertain

Advantages of Memorable Experiences

- Through posterior approximations, the criteria to categorize examples naturally emerges
 - Generalizes existing concepts such as support vectors, influence functions, inducing inputs etc
- Local parameters are available for free and applies to almost "any" ML problem
 - Supervised, unsupervised, RL
 - Discrete/continuation loss and model parameters
- The sensitivity of posterior leads to "Bayes Duality"

The Bayes-Duality Project

Toward AI that learns adaptively, robustly, and continuously, like humans







Emtiyaz Khan

Research director (Japan side)

Approx-Bayes team at RIKEN-AIP and OIST

Julyan Arbel

Research director (France side)

Statify-team, Inria Grenoble Rhône-Alpes

Kenichi Bannai

Co-PI (Japan side)

Math-Science Team at RIKEN-AIP and Keio University

Rio Yokota

Co-PI (Japan side)

Tokyo Institute of Technology

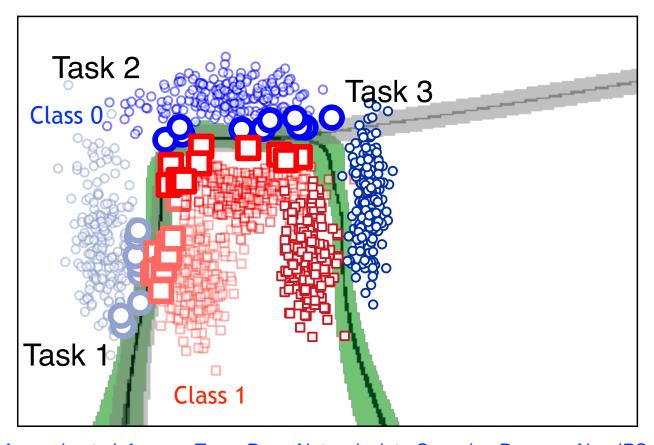
Received total funding of around USD 3 million through JST's CREST-ANR and Kakenhi Grants.

Adaptation

Continual Learning without forgetting the past (by using memorable examples)

Continual Learning

Avoid forgetting by using memorable examples [1,2]



- 1. Khan et al. Approximate Inference Turns Deep Networks into Gaussian Process, NeurIPS, 2019
- 2. Pan et al. Continual Deep Learning by Functional Regularisation of Memorable Past, NeurIPS, 2020

Functional Regularization of Memorable Past (FROMP) [4]

Previous approaches used weight-regularization [1,2]

$$q_{new}(\theta) = \min_{q \in \mathcal{Q}} \mathbb{E}_{q(\theta)}[\ell_{new}(\theta)] - \mathcal{H}(q) - \mathbb{E}_{q(\theta)}[\log q_{old}(\theta)]$$
 New data Weight-regularizer

We replace it by a functional regularizer using a "Gaussian Process view" of DNNs [2]

$$[\sigma(\mathbf{f}(\theta)) - \sigma(\mathbf{f}_{old})]^{\top} K_{old}^{-1} [\sigma(\mathbf{f}(\theta)) - \sigma(\mathbf{f}_{old})]$$

Kernels weighs examples / according to their memorability

Forces network-outputs to be similar

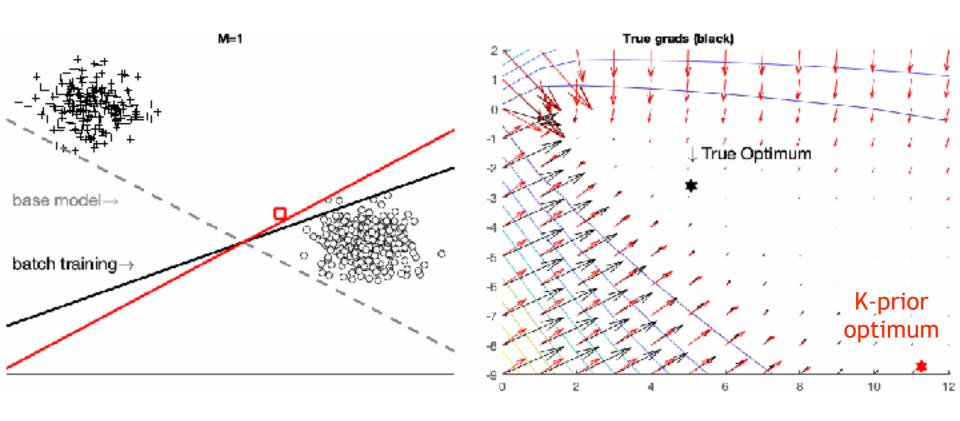
 $\mathbb{E}_{\tilde{q}_{\theta}(\mathbf{f})}[\log \tilde{q}_{\theta_{old}}(\mathbf{f})]$

- 1. Kirkpatrick, James, et al. "Overcoming catastrophic forgetting in neural networks." PNAS 2017
- 2. Nguyen et al., Variational Continual Learning, ICLR, 2018
- 3. Khan et al. Approximate Inference Turns Deep Networks into Gaussian Process, NeurIPS, 2019
- 4. Pan et al. Continual Deep Learning by Functional Regularisation of Memorable Past, NeurIPS, 2020

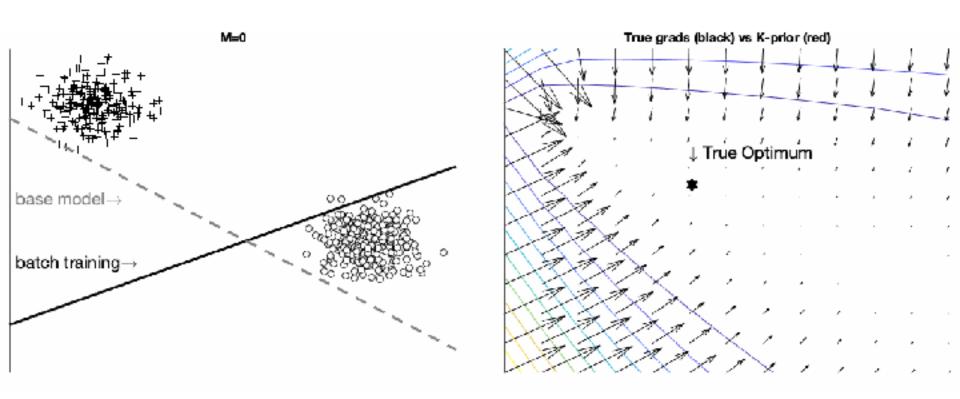
K-Priors and Bayes-Duality

- Dual parameterization of DNNs
 - expressed as Gaussian Process [1]
 - Found using the Bayesian learning rule
- The functional regularizer can provably reconstruct the gradient of the past faithfully [2]
 - Knowledge-Adaptation priors (K-priors)
 - There is a strong evidence that "good" adaptive algorithms must use K-priors

Faithful Gradient Reconstruction



Faithful Gradient Reconstruction



No labels required, so \mathcal{M} can include any inputs!

Summary

- Bayesian principles
 - To unify/generalize/improve learning-algorithms
 - By computing "posterior approximations"
- Bayesian Learning rule (BLR)
 - Derive many existing algorithms
 - Deep Learning (SGD, RMSprop, Adam)
 - Design new algorithms for uncertainty in DL
- Impact: Everything with the same principle

Approximate Bayesian Inference Team

https://team-approx-bayes.github.io/



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Pierre Alguler
Research Scientist



<u>Hugo Monzón</u> Maldonado Postdoc



Happy Buzaaba Postdoc



Erik Daxberger Remote Collaborator University of Cambridge



Paul Chang Remote Collaborator Aalto University



Gian Maria Marconi Postdoc



Thomas Möllenhoff Postdoc



Lu Xu Postdoc



Jooyeon Kim Postdoc



Alexandre Piché
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Geoffrey Wolfer Postdoc



Wu Lin PhD Student University of British Columbia



Peter Nickl Research Assistant



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