On the Construction of Global Estimate from the Local Estimates in Track Fusion

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Abstract

In this paper, we ask the following question for a single target and two pursuers track fusion case: can we obtain the global estimate from the local estimate by simple addition? We observe that the common process noise leads to correlated observations in track-to-track fusion. Because of the correlation the estimation problem cannot be solved with simple addition of the local estimates. It is illustrated for a simple case that an orthogonalization procedure may solve the problem.

1 The Model

Let us consider single target T following the model:

$$\mathbf{x}_{t+1} = A_t \mathbf{x}_t + \mathbf{w}_t \tag{1}$$

where $\mathbf{x}_t \in \Re^n$ is the state vector, $\mathbf{w}_t \in \Re^n$ is the state noise vector. We assume that the initial state vector and the noise vector are *i.i.d* Gaussian random variables, $\mathbf{x}_0 \sim N(\mu_0, \Sigma_0)$, $\mathbf{w}_t \sim N(0, Q_t)$, where, Σ_0 and Q_t are symmetric, positive definite matrices. We have two pursuers, P_1 and P_2, each with a measurement model

$$\mathbf{y}_{t,i} = C_{t,i}\mathbf{x}_t + \mathbf{v}_{t,i}, \quad i = 1,2 \tag{2}$$

$\mathbf{y}_{t,1}, \mathbf{y}_{t,2} \in \Re^m$ are the output vectors, and $\mathbf{v}_{t,1}, \mathbf{v}_{t,2} \in \Re^m$ are the measurement noise vector, with $\mathbf{v}_{t,1} \sim N(0, R_{t,1})$ and $\mathbf{v}_{t,2} \sim N(0, R_{t,2})$. For simplicity, $E(\mathbf{v}_{t,1}\mathbf{w}_t^T) = 0$, $E(\mathbf{v}_{t,2}\mathbf{w}_t^T) = 0$, $E(\mathbf{v}_{t,1}\mathbf{v}_{t,2}^T) = 0$, $E(\mathbf{x}_0\mathbf{w}_t^T) = 0$, $E(\mathbf{x}_0\mathbf{v}_{t,1}^T) = 0$ and $E(\mathbf{x}_0\mathbf{v}_{t,2}^T) = 0$, where $E(\,)$ is the *Expectation* operator. We also assume that the matrix pair, $\{A_t, Q_t^{1/2}\}$ is controllable, and each of $\{A_t, C_{t,i}\}$ is observable. This ensures stability of the Kalman filter, *i.e.*, the error covariance and update gain matrices converge asymptotically to their steady-state values.

2 Global from Local

We denote the observation sequence from i^{th} pursuer as $Y_{s,i} \equiv \{\mathbf{y}_{1,i}, \ldots, \mathbf{y}_{s,i}\}$ for $i = 1,2$. Also the joint observation is denoted by $Y_s \equiv \{Y_{s,1}, Y_{s,2}\}$. We call $E(\mathbf{x}_t|Y_t)$ as the "global" linear minimum mean square error (l.m.s.e.) estimate, in contrast to the $E(\mathbf{x}_t|Y_{t,i})$ as the local estimate of i^{th} pursuer. As stated in (Kailath *et al.* 2000), the global estimate using the local estimates from the pursuers can be found as,

$$E(\mathbf{x}_t|Y_t) = E(\mathbf{x}_t|Y_{t,1}) + E(\mathbf{x}_t|Y_{t,2}) \tag{3}$$

if and only $E(\mathbf{y}_{i,1}, \mathbf{y}_{i,2}) = 0$ for $i = 1, \ldots, t$. However for the model given in Section 1,

$$\begin{aligned} E(\mathbf{y}_{i,1}\mathbf{y}_{j,2}) &= E[(C_{i,1}\mathbf{x}_i + \mathbf{v}_{i,1})(C_{j,2}\mathbf{x}_j + \mathbf{v}_{j,2})] & (4)\\ &= E[C_{i,1}\mathbf{x}_i\mathbf{x}_j C_{j,2}^T + \mathbf{v}_{i,1}\mathbf{v}_{j,1}^T] & (5)\\ &= C_{i,1}E(\mathbf{x}_i\mathbf{x}_j)C_{j,2}^T & (6) \end{aligned}$$

as $\mathbf{v}_{i,1} \perp \mathbf{v}_{j,2} \perp \mathbf{x}_0$. Using the state equation in the state-space model, we have,

$$E(\mathbf{x}_i\mathbf{x}_j) = \begin{cases} A^{i-j}R_{x,j}, & i \geq j \\ R_{x,j}A^{(i-j)T} & i \leq j \end{cases} \tag{7}$$

which gives,

$$E(\mathbf{y}_{i,1}\mathbf{y}_{j,2}) = \begin{cases} C_{i,1}A^{i-j}R_{x,j}C_{j,2}^T, & i \geq j \\ C_{i,1}R_{x,j}A^{(i-j)T}C_{j,2}^T & i \leq j \end{cases} \tag{8}$$

Easy to see that the correlation will not be zero, and hence the estimation using the joint observations cannot be found using a simple addition with Eq. (3). To apply this equation one needs to remove the correlation among the observations, which we illustrate with simple example. Consider two observations y_1 and y_2 associated with a common state x, then the component in y_2 orthogonal to y_1 can be found as,

$$z = y_2 - \frac{\langle y_1 y_2 \rangle}{\langle y_1 y_1 \rangle} y_1 \tag{9}$$

where $\langle , \rangle$ is inner-product in the vector-space of random variables (which is expectation, see (Kailath *et al.* 2000), page 328 for details). Then Eq. (3) can be used to find the estimate,

$$E(x|y_1, y_2) = E(x|y_1) + E(x|z) \tag{10}$$

Definitely this equation cannot be easily written in terms of the local estimates.

References

Kailath, T., A. H. Sayed and B. Hassibi (2000). *Linear Estimation*. Prentice Hall.