Decentralized Information Filter

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Abstract

In this paper we describe the decentralized Information filter (DIF). We consider a single target and multi-pursuer case with full-rate point-to-point communication links and present the possible schemes with a DIF algorithm. Several important points in comparison with measurement fusion (MF) have been discussed. We observe that the choice of MF and DIF involves a trade-off between the computational and communication requirement; DIF has less computational requirement, but more communication requirement.

1 The Model

Consider a field of action with a single target T following the model:

$$\mathbf{x}_{t+1} = A_t \mathbf{x}_t + \mathbf{w}_t \tag{1}$$

where $\mathbf{x}_t \in \Re^n$ is the state vector, $\mathbf{w}_t \in \Re^n$ is the state noise vector. We assume that the initial state vector and the noise vector are *i.i.d* Gaussian random variables, $\mathbf{x}_0 \sim N(\mu_0, \Sigma_0)$, $\mathbf{w}_t \sim N(0, Q_t)$, where, Σ_0 and Q_t are symmetric, positive definite matrices. We consider N pursuers, $P_1, \ldots, P_N$, each with a measurement model

$$\mathbf{y}_{t,i} = C_{t,i}\mathbf{x}_t + \mathbf{v}_{t,i}, \quad i = 1, \ldots, N \tag{2}$$

$\mathbf{y}_{t,i} \in \Re^m$ are the output vectors, and $\mathbf{v}_{t,i} \in \Re^m$ are the measurement noise vectors, with $\mathbf{v}_{t,i} \sim N(0, R_{t,i})$. For simplicity, $E(\mathbf{v}_{t,i}\mathbf{w}_t^T) = 0$, $E(\mathbf{v}_{t,i}\mathbf{v}_{t,j}^T) = \delta_{ij}$, $E(\mathbf{x}_0\mathbf{w}_t^T) = 0$ and $E(\mathbf{x}_0\mathbf{v}_{t,i}^T) = 0$ for all i, j, where $E(\ )$ is the *Expectation* operator. We also assume that the matrix pair, $\{A_t, Q_t^{1/2}\}$ is controllable, and each of $\{A_t, C_{t,i}\}$ is observable. This ensures stability of the Kalman filter, *i.e.*, the error covariance and update gain matrices converge asymptotically to their steady-state values. We denote the observations of i^{th} pursuer as $Y_{s,i} \equiv \{\mathbf{y}_{1,i}, \ldots, \mathbf{y}_{s,i}\}$, and the combined observation from all the pursuers as $Y_s \equiv \{Y_{s,1}, \ldots, Y_{s,N}\}$. Further, we use the following definitions for the state estimates, i.e. conditional expectations of the states and the corresponding error covariances:

$$\textbf{Local estimate :}\ \hat{\mathbf{x}}_{t|s,i} = E\left(\mathbf{x}_t | Y_{s,i}\right) \quad P_{t|s,i} = E\left((\mathbf{x}_t - \hat{\mathbf{x}}_{t|s,i})(\mathbf{x}_t - \hat{\mathbf{x}}_{t|s,i})^T | Y_{s,i}\right) \tag{3}$$

$$\textbf{Global estimate :}\ \hat{\mathbf{x}}_{t|s} = E\left(\mathbf{x}_t | Y_s\right) \quad P_{t|s} = E\left((\mathbf{x}_t - \hat{\mathbf{x}}_{t|s})(\mathbf{x}_t - \hat{\mathbf{x}}_{t|s})^T | Y_s\right) \tag{4}$$

We assume that the pursuers can measure the target position over the whole field of action. In addition each pursuer communicate with all the other pursuer with a point-to-point communication link and the communication is full rate, i.e. communication takes place at each time-step.

2 The Decentralized Information Filter (DIF)

Kalman filter can be written in the information form which is in terms of inverse of error-covariance matrix. Specifically $\hat{Z}_{t|s} = P_{t|s}^{-1}$ and $\hat{\mathbf{z}}_{t|s} = P_{t|s}^{-1}\hat{\mathbf{x}}_{t|s}$, called *information matrix* and *information state vector* are estimated. A detailed description of information filter can be found in (Anderson and Moore 1979),(Khan 2005). Information filter has interesting properties which makes it easy to decouple and decentralize. Here we describe a decentralized information filter for the model described in the previous section. A detailed description can also be seen at (Mutambara 1998),(Durrant-Whyte and Stevens n.d.).

2.1 Local Information Filter

Each pursuer computes a local estimate using the local observations. Denoting the local information estimate as $\tilde{Z}_{t|s,i} = P_{t|s,i}^{-1}$ and $\tilde{\mathbf{z}}_{t|s,i} = P_{t|s,i}^{-1}\hat{\mathbf{x}}_{t|s,i}$, each pursuer runs these equations:

$$\tilde{\mathbf{z}}_{t|t,i} = \tilde{\mathbf{z}}_{t|t-1,i} + \mathbf{i}_{t,i}, \quad \tilde{Z}_{t|t,i} = \tilde{Z}_{t|t-1,i} + I_{t,i} \tag{5}$$

$$\tilde{\mathbf{z}}_{t+1|t,i} = L_{t+1|t,i}\tilde{\mathbf{z}}_{t+1|t,i}, \quad \tilde{Z}_{t+1|t,i} = [A_t \tilde{Z}_{t|t,i}^{-1} A_t^T + Q_t]^{-1} \tag{6}$$

where $L_{t+1|t,i} = \tilde{Z}_{t+1|t,i} A_t \tilde{Z}_{t|t,i}^{-1}$. Also $\mathbf{i}_{t,i} = C_{t,i}^T R_{t,i}^{-1} \mathbf{y}_{t,i}$ and $I_{t,i} = C_{t,i}^T R_{t,i}^{-1} C_{t,i}$, and are called "information vector and matrix associated with the observations $\mathbf{y}_{t,i}$".

2.2 Communication and Assimilation

If informations from all the pursuers are available with the global prediction, the global estimate can be computed as follows:

$$\hat{\mathbf{z}}_{t|t} = \hat{\mathbf{z}}_{t|t-1} + \sum_{j=1}^{N} \mathbf{i}_{t,j}, \quad \hat{Z}_{t|t} = \hat{Z}_{t|t-1} + \sum_{j=1}^{N} I_{t,j} \tag{7}$$

Our objective is to make the local estimate at each pursuer equal to this global estimate, i.e. $\tilde{\mathbf{z}}_{t|t,i} = \hat{\mathbf{z}}_{t|t}$ and $\tilde{Z}_{t|t,i} = \hat{Z}_{t|t}$ for all $i = 1, \ldots, N$.

If we assume the local predictions $\tilde{\mathbf{z}}_{t|t-1,i}$ for all pursuers to be equal to the global prediction, one way to obtain global estimate at each pursuer is to communicate the information vector associated with the observations, i.e. $\mathbf{i}_{t,i}$ and $I_{t,i}$. Then above equation can be used to assimilate the information to get the global estimate. The local estimates can be then equated to this global estimate and the local prediction can be found using this global estimate (and hence the local prediction will be equal to the global prediction so that our assumption is satisfied).

Another alternative is to communicate the local estimate $\tilde{\mathbf{z}}_{t|t,i}$. Again assuming the local prediction at all the pursuers is equal to global prediction, the information present in the communicated local prediction of j^{th} pursuer can be recovered at i^{th} pursuers as follows:

$$\mathbf{i}_{t,j} = \tilde{\mathbf{z}}_{k|k,j} - \hat{\mathbf{z}}_{k|k-1,i} \tag{8}$$

Hence the following equations will make the local estimate equal to the global estimate at all the pursuers:

$$\tilde{\mathbf{z}}_{t|t,i} = \tilde{\mathbf{z}}_{t|t-1,i} + \sum_{j=1}^{N} [\tilde{\mathbf{z}}_{k|k,j} - \tilde{\mathbf{z}}_{k|k-1,i}] \tag{9}$$

$$\tilde{Z}_{t|t,i} = \tilde{Z}_{t|t-1,i} + \sum_{j=1}^{N} [\tilde{Z}_{k|k,j} - \tilde{Z}_{k|k-1,i}] \tag{10}$$

Note that all this formulation will yield global estimates at each pursuer only if there is full rate communication in absence of which the local prediction will not be equal to the global prediction and hence the prediction term will always contain an error making the filter to be suboptimal.

3 Comparison with Measurement Fusion (MF)

In measurement fusion (MF), eq. (1-2) are combined into one state-space model,

$$\begin{aligned} \mathbf{x}_{t+1} &= A_t \mathbf{x}_t + \mathbf{w}_t \\ \mathbf{y}_t &= C_t \mathbf{x}_t + \mathbf{v}_t \end{aligned} \tag{11}$$

where $\mathbf{y}_t = [\mathbf{y}_{t,1}^T, \ldots, \mathbf{y}_{t,N}^T]^T$, $\mathbf{v}_t = [\mathbf{v}_{t,1}, \ldots, \mathbf{v}_{t,2}]^T$ and $C_t = [C_{t,1}^T \ldots C_{t,N}^T]^T$. The measurements are communicated among the pursuers, and an estimate is obtained with a Kalman filter (Kailath *et al.* 2000). Here are some important points:

Algebraic Equivalence: As discussed in (Mutambara 1998),(Khan 2005), every measurement fusion scheme can be equivalently represented by a DIF. This is because of the algebraic equivalence of the information filter and Kalman filter. Hence both of these methods give identical and optimal solutions.

Computational Redundancy In measurement fusion, as the *raw* measurements are communicated, same computations are repeated at all the the pursuers. However in DIF, first a local estimate is computed based on the local measurement and then transmitted to the other pursuers, where it's just added to get the global estimate. Hence the algorithm removes the redundant computation.

Computational complexity In decentralized information filter, there is no gain or innovation covariance matrices and the maximum dimension of a matrix to be inverted is of the state dimension, which is usually smaller than the observation dimensions in a multi-sensor system; $(CP_{t|t-1}C^T + R)$ in Kalman filter and

Table 1: Variables to be communicated in MF and DIF

	MF Variables (size)	IF Variables (size)
Model known	$\mathbf{y}_{t,i}$ $(m \times 1)$	$\mathbf{i}_{t,i}$ or $\tilde{\mathbf{z}}_{t,i}$ $(n \times 1)$
Model Not known	$\mathbf{y}_{t,i}$ $(m \times 1)$ $R_{t,i}$ $(m \times m)$ $C_{k,i}$ $(m \times n)$	$\mathbf{i}_{t,i}$ or $\tilde{\mathbf{z}}_{t,i}$ $I_{t,i}$ or $\tilde{Z}_{t,i}$ $(n + n \times n)$

$(A_t^T Z_{t|t}^{-1} A_t + Q_t^{-1})$ in information filter, see (Anderson and Moore 1979),(Mutambara 1998). In Kalman filter change in number of pursuers involves a huge change in computational requirements, whereas in DIF, it will not affect. (An alternative of this could be to use information filter in measurement fusion instead of a Kalman filter).

Communication requirement Assuming full rate communication communication requirement in each of the pursuer is given in the Table 1. We can see that as in multi-sensor system, usually, dimension of the state will be less than the dimension of the local observations, the communication requirements are higher in DIF.

We can see that for a full-rate communication both DIF and MF are equivalent in algebraic sense, and the trade-off is between the communication requirement and the computational requirement. However when the communication is not full-rate, transmission of estimated state in the DIF will have some advantage over communicating the raw measurement, as the estimated state contains the past informations. Also intuitively there should be a trade-off on the communication requirement and the *information gain* by communicating the estimated state vectors, which needs to be studied.

References

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